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QUANTUM DOTS: A BRIDGE BETWEEN QUANTUM MECHANICS AND ADVANCED TECHNOLOGIES

KROPKI KWANTOWE: POMOST MIĘDZY MECHANIKĄ KWANTOWĄ A ZAAWANSOWANYMI TECHNOLOGIAMI

Abstract

Since the 1980s, it has been observed that spatially confined structures on the order of nanometers exhibit fascinating phenomena, opening new possibilities for research and applications in various scientific fields. Such rigorous spatial confinement results in quantum phenomena like excitations and tunneling playing a dominant role in charge transport. This has opened new research directions in microelectronics that utilize quantum phenomena. These new directions include the ability to create, characterize, and manipulate artificial structures whose features are controlled at the atomic level. They span diverse research areas such as mechanics, engineering, physics, chemistry, materials science, molecular biology, and medicine.

The fundamental components of modern electronic devices utilizing quantum phenomena are structures spatially confined in all three dimensions. Such quasi-zero-dimensional objects are called quantum dots (QDs).

Quantum dots are applied in both fundamental research and practical uses. Spatial confinement allows QDs to be treated as artificial atoms with controllable energy levels. This has proven invaluable in studying many-body phenomena occurring in solid materials, including e.g. strong electronic correlations, interferences, quantum decoherence, and spin effects. In parallel with these studies, quantum dots have also been explored for practical purposes. Their ability to absorb radiation at precisely defined wavelengths has led to the construction of photodetectors combining features such as high sensitivity, fast response time, and tunability. The same

Streszczenie

Już od lat osiemdziesiątych ubiegłego wieku zauważono, że struktury ograniczone przestrzennie do rozmiarów rzędu nanometrów wykazują fascynujące zjawiska, które otwierają nowe możliwości badań oraz zastosowań w wielu dziedzinach nauki. Tak rygorystyczne ograniczenie przestrzenne powoduje, że dominującą rolę w transporcie ładunków odgrywają zjawiska kwantowe takie jak wzbudzenia czy tunelowanie. Otworzyło to nowe kierunki badań nad mikroelektroniką opartą o wykorzystanie zjawisk kwantowych. Te nowe kierunki obejmują zdolność do tworzenia, charakteryzowania i manipulowania sztucznymi strukturami, których cechy są kontrolowane na poziomie atomowym. Dotyczą one różnorodnych dziedzin badawczych, takich jak mechanika, inżynieria, fizyka, chemia, nauki o materiałach i biologia molekularna czy medycyna.

Podstawowy budulec nowoczesnych urządzeń elektronicznych wykorzystujących zjawiska kwantowe stanowią struktury ograniczone przestrzennie we wszystkich trzech wymiarach. Takie kwazizerowymiarowe obiekty nazywane są kropkami kwantowymi.

Kropki kwantowe wykorzystywane są zarówno w badaniach podstawowych, jak również przy zastosowaniach praktycznych. Ograniczenie przestrzenne powoduje, że QD mogą być traktowane jako sztuczne atomy o kontrolowalnych poziomach energetycznych. Okazało się to nieocenione przy badaniach zjawisk wielociałowych występujących również w materiałach litych. Silne korelacje elektronowe, interferencje, kwantowe dekoherencje czy efekty spinowe to tylko kilka przykładów. Równolegle do tych prac

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properties have enabled quantum dot technologies to be applied even in domestic environments. Quantum dot-based light-emitting diodes with well-defined wavelengths have been used to create QLED displays with unprecedented resolutions. In medicine, quantum dots are employed as tumor markers that selectively bind to cancer cells. Quantum dots also serve as the basic units in single-electron transistors.

This paper provides a comprehensive overview of the basic physics governing quantum dots, their unique properties, and the diverse range of applications stemming from their quantum confinement effects.

Keywords: Quantum Dots, Condensed matter

kropki kwantowe były również badane pod kątem praktycznym. Zdolność do absorbowania promieniowania o ściśle określonych długościach fal spowodowała, że kropki kwantowe wykorzystano do konstrukcji fotodetektorów łączących w sobie cechy takie jak wysoka czułość, szybki czas reakcji czy możliwość dostrajania. Te same własności spowodowały, że technologie oparte o kropki kwantowe znalazły zastosowanie nawet w domowych warunkach. Diody emitujące światło o bardzo dobrze określonej długości fal wykorzystano do tworzenia ekranów QLED o nieosiągalnej dotychczas rozdzielczości. W medycynie kropki kwantowe wykorzystuje się jako markery nowotworowe, które selektywnie wiążą się z komórkami nowotworowymi. Kropki kwantowe stanowią również podstawową jednostkę w jednoelektronowych tranzystorach.

Niniejszy artykuł zawiera kompleksowy przegląd podstawowej fizyki rządzącej kropkami kwantowymi, ich unikalnych właściwości i różnorodnego zakresu zastosowań wynikających z ich dyskretnych poziomów energetycznych.

Słowa kluczowe: kropki kwantowe, skondensowana materia

1. SPATIAL CONFINEMENT – DISCRETE ENERGY DISTRIBUTION

Materials at the nanoscale often exhibit characteristics that are intermediate between what we observe in a macroscopic solid state and what can be observed in an atomic or subatomic system. Consider a crystal constructed from several dozen or even several hundred atoms. On one hand, its characteristics will differ from those typical of a single atom or molecules consisting of a small number of atoms. On the other hand, we cannot assume that its properties will be identical to those observed in macroscopic objects containing billions of atoms. In such structures, valence electrons are not bound to a specific atom (electrons are delocalized/shared), which is characteristic of macroscopic bodies, but on the other hand, their discrete energy distribution means that their description escapes the standard model of band structure. Quantum dots thus represent a new class of materials whose properties cannot be described either purely at the atomic level or at the macroscopic level.

In macroscopic materials, electrons have a continuous distribution of available energies arranged in energy bands. These bands arise from the overlapping of atomic orbitals in the crystal lattice. However, if we spatially limit such a metal, or semiconductor (in all three dimensions), the number of available states is limited and the distribution of energies available to electrons becomes discrete (similar to what happens in isolated atoms).

Quantum dots are structures composed of a few to several thousand atoms arranged in such a way that in each of the three spatial dimensions, they are limited to sizes on

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the order of a few to several tens of nanometers. They thus represent a class of problems related to particles in so-called potential wells. According to de Broglie's hypothesis, each particle is associated with a wave of length $\lambda = h/p$, where h is Planck's constant, and p is the momentum of the particle. The shape of the wave function for a given particle depends on the environment (mainly the size of the well) in which it is located and is described by the Schrödinger equation. Solving this problem indicates that a particle confined in a potential well can only have energies for which a standing wave is formed inside the well (Fig. 1). This leads to the so-called quantization of energy levels. A particle in a well can only have certain discrete energy levels (states) dependent on the size of the well. Individual states are separated from each other by an energy gap ΔE. This gap is inversely proportional to the square of the size of the well ($\Delta E \sim 1/L^2$). Hence, by controlling the size of the well (quantum dot), the energy gap can be adjusted. Quantum dots of relatively large sizes have densely "packed" states with small forbidden energies, for small dots the energy separation between levels is significantly larger.

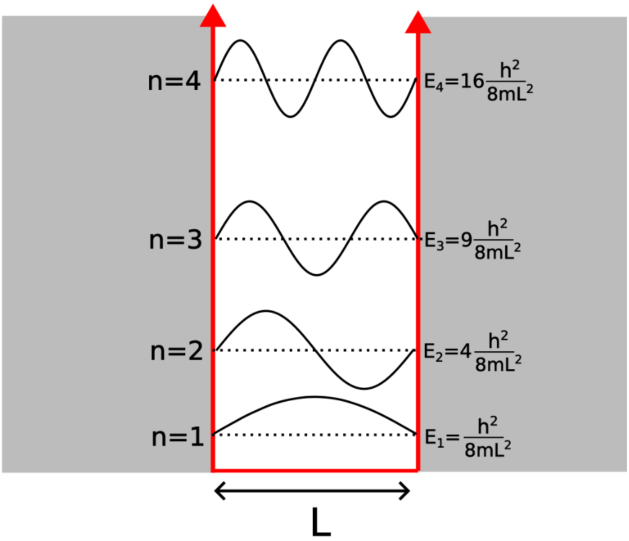

Fig. 1. Wave function of electron placed in 1D infinite quantum well

Source: LibreTexts Physics. (n.d.), Quantum Mechanics: The Infinite Potential Well [Diagram]. In UCD: Physics 7C - General Physics. Retrieved from https://phys.libretexts.org/Courses/University_of_California_Davis/UCD%3A_Physics_7C_-_General_Physics/9%3A_Quantum_Mechanics/9.4%3A_The_Infinite_Potential_Well [accessed: 16.02.2024].

Early works focusing on the theory and manufacturing of structures spatially limited to the extent that quantum effects play a dominant role appeared as early as the 1960s. Methods based on evaporation and deposition of substrate atoms on the surface of a substrate allowed for the production of ultra-thin films (surfaces) with thicknesses ranging from a few to several tens of atomic layers[1, 2]. The atomic surfaces obtained in this way can be treated as a structure in which electrons can move only in two dimensions. Initial work on thin films focused mainly on the analysis of materials exhibiting superconducting properties at low temperatures. It was noted that spatial limitation causes a change in the conditions for phase transition from the normal state to the superconducting state. Among other findings, it was discovered that ultra-thin layers made of superconducting materials have a significantly higher temperature of transition to the superconducting state than their macroscopic counterparts[3]. Although initial studies focused mainly on superconductors, researchers' interest quickly expanded to other types of ultra-thin films, including those exhibiting magnetic properties[4, 5]. Work on magnetic anisotropy and rapid, controllable reversibility of magnetization laid the groundwork for the construction of fast memory units[6]. The discovery of the Tunnel Magnetoresistance (TMR) phenomenon between thin layers of ferromagnets, in turn, paved the way for the construction of more efficient and miniaturized mass storage devices, such as MRAM (Magnetoresistive Random-Access Memory)[7]. The computer industry quickly recognized the potential associated with ultra-thin films, and in 1968, IBM used them to develop ultrafast thin-film memories[8]. Another commercial technological leap using ultra-thin layers of ferromagnets occurred in the 1990s when IBM presented a prototype MRAM memory that used the magneto-resistance phenomenon for data storage.

The introduction of more advanced methods (mainly in the 80s) such as molecular beam epitaxy[9] (MBE), atomic layer deposition[10] (ALD), or the self-assembled

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structures method[11] led to increased precision in the deposition of ultra-thin layers, up to the atomic scale, and also enabled the creation of structures limited spatially in the remaining dimensions. The creation of one-dimensional and finally zero-dimensional objects became a milestone opening up a new field of science, namely nanotechnology. The behavior of electrons in one-dimensional structures, such as nanowires or carbon nanotubes, does not follow the standard description of metals based on the Fermi liquid concept. The understanding of the key role of electron interactions in one-dimensional chains, made by Luttinger and Tomonaga[12], led to the development of an alternative description known as Luttinger liquid. This description takes into account the collective behavior of electrons in such structures. Further experimental work allowed for the creation of atomic islands limited in all three dimensions. Groundbreaking were the studies by C. B. Murray[13], which presented a relatively simple method for creating micro-islands with diameters on the order of nanometers from cadmium compounds (CdSe, CdTe, and CdS). This process enabled the production of uniform particles with diameters from 12 to 115 angstroms (1A = 10^-10m) in one reaction, in macroscopic quantities. The produced particles were characterized by uniform size and shape while demonstrating properties of absorption and emission of light at well-defined frequencies even at room temperature. These zero-dimensional structures exhibiting a discrete energy spectrum were named quantum dots[14]. Figure 2 shows the dependence of the density of states on the energy of electrons for solid materials (3D) and those limited spatially in one, two, and three dimensions. The synthesis of quantum dots proved to be such a significant achievement that, by decision of the Swedish Academy of Sciences in 2023, the Nobel Prize in Chemistry was awarded for the discovery and synthesis of quantum dots[15].

Contemporary experimental techniques not only allow for precise creation of objects at almost atomic scale or placing them on various surfaces (e.g., superconductors or ferromagnets) but also their combination into specific configurations chosen by researchers. Thanks to these advanced techniques, “hybrids” of nanoobjects or even nanodevices such as the SQUID interferometer can be created. Junctions containing connected quantum dots in various configurations, dots connected with nanowires or graphene, and implementations where quantum dots are connected to single-atom topological chains are being created. The possibilities of such nanodevices go

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beyond the investigation of known phenomena, as they allow for the creation of new phases and electronic states not present in solid materials. Utilizing these new capabilities in quantum communication may open new horizons in technological applications and computational concepts, contributing to the development of advanced quantum cryptography and information processing systems using quantum bits, increasing the security and efficiency of data transmission.

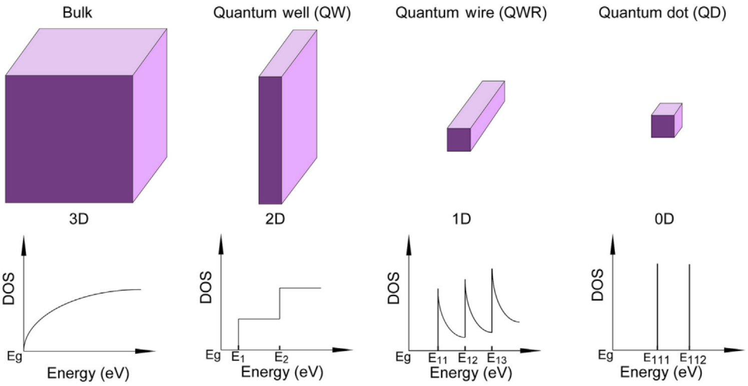

Fig. 2. Density of States Diagram in Structures: Three-Dimensional (first panel from the left), Two-Dimensional Surface (second from the left), One-Dimensional Wire (third from the left), and Quantum Dot (fourth from the left)

Source: K. Tomioka, T. Fukui, Growth of Semiconductor Nanocrystals. In Handbook of Crystal Growth (pp. 749–793). Elsevier. (2015). https://doi.org/10.1016/b978-0-444-56369-9.00018-6.

3. QUANTUM DOTS – ARTIFICIAL ATOMS IN FUNDAMENTAL RESEARCHES

Quantum dots, since their synthesis began, have been the subject of extensive fundamental research, both experimentally and theoretically. Early works were primarily focused on the production and characterization of the dots themselves and their optical properties. One of the basic issues was the quantum size effect leading to the quantization of energy levels. Measurements of the energy structure of dots were made indirectly through the analysis of the absorption (or emission) spectrum and its dependence on the size of the quantum dot[16]. However, it quickly became apparent that quantum dots offer an incredibly broad spectrum of research possibilities that extend beyond the initial interests. Quantum dots play a key role as unique tools enabling a deeper understanding of phenomena occurring at the microscopic level. Due to their properties, they are often referred to as a kind of “artificial atoms” or artificial impurities. Unlike traditional impurities, quantum dots allow for relatively easy control over properties such as energy levels or hybridization with electrodes (environment). Quantum dots, like single atoms, have discrete energy levels, but the

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filling of these levels can be modified by applying a gate voltage. Charge transport in junctions containing QD occurs through the states available on the studied object. Therefore, analyzing the current flowing through such junctions allows insight into the energetic structure of the artificial impurity. The controllability and access to information about available energy levels make QDs extremely useful in analyzing processes occurring in impurities and also offer an excellent platform for confronting theoretical predictions with experiments. These properties of quantum dots open a wide spectrum for fundamental research. Quantum dots are used in various areas of condensed matter physics research. Below are some of the research directions that provide not only a deeper understanding of microscopic processes but also open new perspectives for future technologies. Among the most widely discussed are:

1. Coulomb blockade – manifests as a prohibition of additional electrons passing through a quantum dot when it is already occupied by another electron. This effect was utilized in the construction of a single-electron transistor, whose general operating principle will be discussed in the next paragraph[17].
2. Kondo effect – a phenomenon that results in an increase in the electrical conductivity of a quantum dot at low temperatures, caused by the interaction of electrons' spins in the dot with electrons in the electrodes. In the Kondo resonance, the spin of an electron localized on the quantum dot is screened by exchange interactions with delocalized electrons from the metal. The coexistence of the Kondo effect with other many-body effects provides a wide spectrum of research[18].
3. Inducing local superconducting electron pairs – One of the key mechanisms for the formation of the superconducting state in macroscopic materials is the pairing of electrons into electron pairs (Cooper pairs). As a result of the pairing of a huge number of electrons, a so-called condensate is formed - large groups of particles occupying the same state. A group of pairs behaving like one superparticle. If a quantum dot is connected to a superconductor, local electron pairs can be generated in its area. Although the dot is not a superconductor, studies on local pairs and their mutual relations with Coulomb interactions provide invaluable information about the nature of electron pair formation and their interactions with impurities[19].
4. Andreev reflections – In systems where a quantum dot is located between a metal and a superconductor, a very non-trivial mechanism of charge transfer is also implemented. Since the charge carriers in a superconductor are electron pairs and there are no single-particle states near the Fermi surface, a single electron cannot propagate in them. At metal-superconductor junctions, a single

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[19] Ibidem; J. Barański, T. Domański, In-gap states of a quantum dot coupled between a normal and a superconducting lead. In Journal of Physics: Condensed Matter (Vol. 25, Issue 43, p. 435305). IOP Publishing. (2013). https://doi.org/10.1088/0953-8984/25/43/435305.

electron from the metal with energy above the Fermi energy can be converted into an electron pair propagating in the superconductor. To conserve charge, such an electron must "pick up" another electron with energy lower than the Fermi surface. As a result, in the metal, a particle is converted into a hole (lack of electron below the Fermi surface). A quantum dot squeezed between such two electrodes mediates these processes. Thanks to the controllable nature of quantum dots in described junctions, nuances of such exchange can be better understood[20].

5. Quantum entanglement – In systems where two or more quantum dots are connected, due to exchange interactions between electrons of both dots, it is possible to create local entangled states. States in which two particles are linked in such a way that measuring one of them immediately affects the state of the other. Entangled states form the basis for the most sophisticated quantum technologies like teleportation or quantum cryptography[21].
6. Quantum interference – In systems where electric charge transport can take different paths, such as a setup of two parallel quantum dots placed between two electrodes, phenomena related to the wave nature of matter occur. Electrons transmitted through such a system do not move independently but as matter waves, which can overlap each other. As a result, when these waves meet, quantum interference occurs – amplifying or weakening the probability of electrons' transmission through the system, depending on their phase shift[22, 23].
7. Decoherence – When a nanoscopic system is prepared in such a way that it exhibits non-trivial quantum states, such as superposition or entanglement, interactions with the environment can destroy these superpositions. Interaction with the environment can act like a measurement effectively destroying quantum entanglement[24, 25, 26]. Preventing decoherences is one of the main challenges in realizing stable computational systems based on quantum logic. Since the above-mentioned quantum interferences are also subject to these rules, analysis of quantum interferences can answer how well the system is isolated from decoherence.
8. Detection, analysis, and manipulation of topological states – In the context of the resistance of quantum states to decoherence discussed above, new perspectives

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have emerged in recent years. On one-dimensional monoatomic chains placed on the surface of a superconductor, so-called Majorana quasi-particles have been realized. These are single particles found in a non-local superposition. The non-locality of Majorana quasi-particles means that these states are resistant to local decoherences. One of the fundamental features of quantum dots, which makes them irreplaceable in these studies, is the leakage effect, involving the controlled exchange of states between the dot and the environment[27, 28]. This allows for the detection of topological states (such as Majorana quasi-particles), enabling not only confirmation of their existence but also the study of their properties and dynamics[29]. Moreover, theoretical work suggests that quantum dots may provide an advanced platform for manipulating these states[30].

9. Quantum logic gates – Quantum dots are considered to be natural equivalents of logic gates in quantum logic[31, 32, 33]. Unlike classical bits where a bit is represented by |0> or |1>, in quantum logic a qubit is represented by a superposition α|0> + β|1>, where $|\alpha|^2$ and $|\beta|^2$ determine the probability of being in state 0 or 1 (until the moment of measurement, the qubit is simultaneously zero and one). The basic gates in quantum logic are the CNOT gate and the Hadamard gate. The CNOT gate causes a swap of the probabilities of zeros and ones. The state α|0> + β|1> after passing through the CNOT gate changes to α|1> + β|0>. The Hadamard gate, in turn, changes the so-called phase of a qubit represented by a "-" sign at zero or one. In this case, the state α|0> + β|1> is converted to the state α|0> - β|1>. The CNOT and Hadamard gates are used, among others, in quantum cryptography[34]. Nowadays, these logic gates are implemented in coupled quantum dot systems. Experimental implementations usually involve the manipulation of spin states of single electrons where one spin (e.g., up)

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corresponds to the logical value 0, while the other (down) corresponds to 1. In the work[35], electrons located on two quantum dots were in superposition of these states, which realizes a qubit. By magnetically controlled change of exchange interactions between electrons of both dots, researchers made changes to the orientation of individual spins, implementing the CNOT gate.

4. SINGLE ELECTRON TRANSISTOR – COULOMB BLOCKADE

One of the milestones in nanoelectronics is the construction of a new type of transistor that allows for the controlled flow of individual electrons, known as the single electron transistor (SET)[36]. The concept, based on exploiting the Coulomb blockade phenomenon to construct such devices, emerged in the 1980s[37]. The basic unit of such a device is a quantum dot placed between two electrodes. Electrons are transferred between the electrodes (from source to drain) via quantum tunneling. The specificity of quantum dots allows for controlling the current flow in such a way that single electrons are sequentially transmitted in the circuit. Due to their small sizes, Coulomb interactions play a very significant role in quantum dots. Such interaction can effectively block the transfer of electrons in the junction. Imagine a junction in which a quantum dot is placed between two metallic electrodes (S and D in figures 2a and 2b) as shown in figure 2a. The metallic electrodes have a continuous continuum of states. In the metal, all states below the energy called Fermi energy (Fermi energies for S and D are marked as red lines in figs 2a and 2b) are occupied by electrons, while (at low temperature) all levels above remain empty. If a quantum dot with discrete energy levels is placed between these two electrodes, all its levels below the metal's Fermi surface will be filled as a result of electrons tunneling from the metal. States above the Fermi surface will remain empty. If a voltage V is applied between the S and D electrodes (causing a shift in the metal levels by energy eV), electrons will tend to flow from S to D. Such flow in the discussed junction must occur through the quantum dot.

Consider one state of the dot (ε) which is below the Fermi surface of both metal L and R. The Pauli exclusion principle states that two electrons cannot be in the same quantum state. Therefore, an electron on the dot cannot tunnel to the metal because all states in the metal are already occupied. On the other hand, for an additional electron from metal L to tunnel into the dot area, it must overcome the Coulomb barrier – energy must be provided that overcomes the repulsion between electrons. As long as this energy is not provided, the current flow is halted. However, if a voltage (gate voltage) is applied to the dot such that the doubly occupied level

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is between the potentials of S and D, electrons from metal S will be able to pass to metal D via the higher level. Tunneling in this case occurs sequentially – when one electron tunnels from S to QD, the state (ε + U) becomes occupied, so the next electron from S cannot tunnel into the dot area. Only when an electron from QD is tunneled to the D area, the next electron from S can take its place. By controlling the gate voltage, we can thus cause sequential tunneling of individual electrons, which corresponds to a single-electron tunneling transistor. An STM image (Scanning Tunneling Microscope) of the realization of this type of junction is shown in figure (fig. 3c)[38]. The obtained current-voltage characteristic shows a stepwise increase in current intensity depending on the applied voltage (fig. 3d).

Although single-electron transistors are still at the stage of basic research and do not have commercial applications (mainly due to the requirement for low operating temperatures), they are used in fundamental research primarily as precise electric charge meters and electrometers and for analyzing quantum interference effects. One of the applications of SET is also precise radiation measurement.

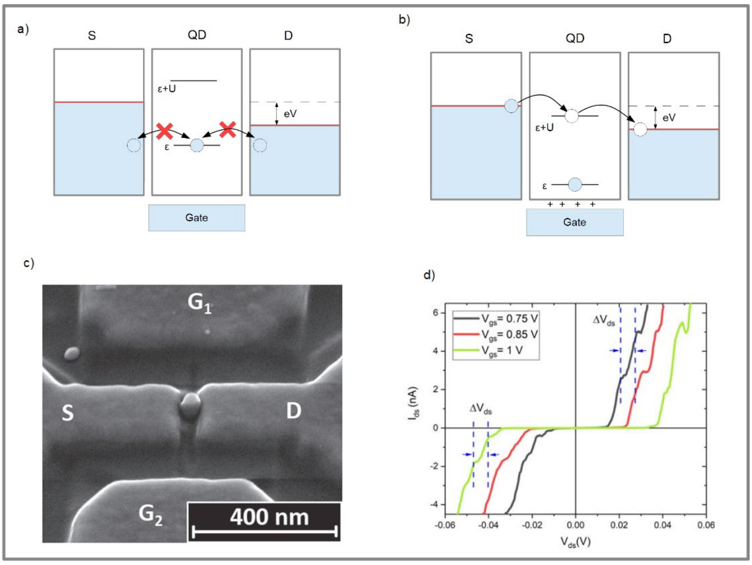

Fig. 3. Panels (a) and (b) present a schematic overview showing the operating principle of a Single Electron Transistor (SET). Without gate voltage, the level below the Fermi surface is occupied, and charge transport is forbidden (a). Gate voltage causes the energy levels to shift, enabling charge transfer (b). Panel (c) shows an STM image from the implementation of a single-electron transistor. Panel (d) displays the obtained current-voltage characteristic of the junction as a function of source-drain voltage (V_{ds}) for three different gate voltages (V_{gs})

Source: G.D. Prima, R. Sachser, P. Trompenaars, H. Mulders, M. Huth, Direct-write single electron transistors by focused electron beam induced deposition. In Nano Futures (Vol. 3, Issue 2, p. 025001). IOP Publishing. (2019). https://doi.org/10.1088/2399-1984/ab151c.

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5. ABSORPTION AND EMISSION OF LIGHT IN QUANTUM DOTS

Research on the absorption and emission of light by quantum dots began in the 1970s, with the aim of creating new electronic and optical devices. At that time, few people anticipated their application in biology or medicine. Quantum dots exhibit unique light absorption and emission properties, which are directly related to their size. This phenomenon allows for the control of the intensity as well as the range of light wavelengths they emit, opening up a wide spectrum of applications for them – from medical diagnostics, through monitoring changes at the cellular level, to applications in optics and electronics.

The interaction of electromagnetic waves with quantum dots can significantly affect their energy distribution. Consequently, if a quantum dot is exposed to radiation of a specific wavelength, it can influence its charge transfer capability. This property is the basis for the construction of photodetectors based on quantum dots[39, 40, 41]. When a quantum dot is subjected to radiation, electron-hole pairs (so-called excitons) are formed in its area. A hole propagating in the S electrode and an electron propagating in D contribute to the current induced by electromagnetic waves. As a result, such radiation can be detected. The general idea of the operation of such detectors is shown in Figure 4.

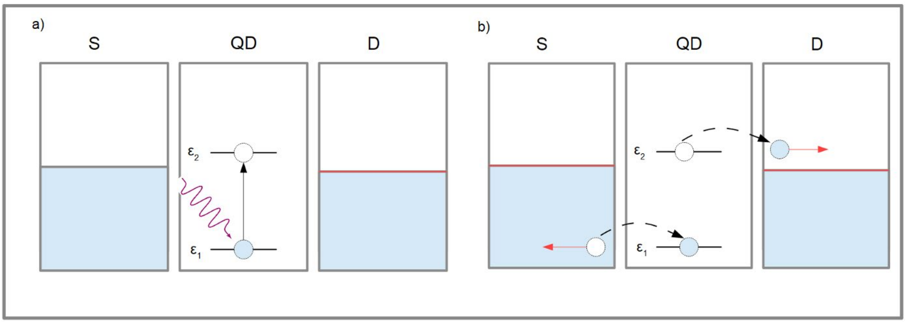

Fig. 4. The operating principle of a quantum dot-based photodetector. (a) A quantum of light causes the transfer of an electron from the ground level (ε1) located below the Fermi surface to the excited state (ε2). The excited electron tunnels to the D area, while an electron from the S metal tunnels to the ε1 position, leaving behind a hole in S (an absence of an electron below the Fermi surface). Since the electron and hole have opposite charges, they move in opposite directions

In the photodetector, quantum dots serve as the active material that absorbs incoming photons. If light composed of photons with energy equal to the difference

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between the highest energy level of the dot (in Figure 4, energy ε_1) below the Fermi surface of the emitter and the level (ε_2 in Figure 4) above the Fermi surface of the collector interacts with an electron at level ε_1, then this electron can absorb the photon's energy. As a result, it will occupy level ε_2 (the electron will be in what's called the excited state), while level ε_1 will remain temporarily unoccupied. Subsequently, the excited electron can tunnel to the collector, while an electron from the emitter with energy equal to ε_1 can tunnel to the unoccupied state ε_1 in the quantum dot. In the electrode, after the tunneled electron, a so-called hole remains (formally, it is the absence of an electron in the metal below the Fermi surface). As a result, we obtain an electron above the Fermi surface and a hole below this surface. Such a state is called an exciton. It is a quasiparticle representing the quantum-mechanical coupling of an electron in the conduction band and a hole (an empty state in the valence band). The electron and hole have opposite charges and move in opposite directions. Detection of such excitations gives a signal that the quantum dot has been irradiated with an electromagnetic wave of precisely defined frequency. Radiation detectors based on QDs, due to their wide range of light absorption and emission, are used for detecting radiation from infrared to X-ray radiation.

Besides detectors, quantum dots also serve as highly efficient light emitters. This feature has led to QDs already being used in practical as well as commercial applications. Quantum dots are used as biomarkers in medical imaging, allowing for precise tracking of biological processes at the cellular level. Their unique properties, such as high brightness, stability, and the ability to be tailored for light emission at specific wavelengths, make them ideal for use in imaging diagnostics. QDs are used in targeted therapy, where they reach diseased tissues or cancer cells, enabling their identification[42]. From a commercial usage perspective, the most advanced product utilizing QDs are QLED (Quantum Dot Light Emitting Diode) displays[43,44,45], which are already available in widely accessible monitors and TV sets. The operating principle of QLED can be characterized as follows: Quantum dots are excited by an electric current, resulting in the absorption of energy and the excitation of an electron from the valence band to the conduction band, creating an electron-hole pair. Then the electron can return to the valence band, recombining with the hole. This process is associated with the emission of a photon (light) with energy equal to the difference between the energy levels of the conduction band and the valence band[46]. Quantum dots are capable of generating light at very precisely defined wavelengths. As a result, screens based on QLED technology produce images that are more vivid and realistic. QLEDs

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also achieve greater brightness with lower energy consumption than traditional LCD screens. Additionally, the very short response time means that dynamically changing images are smoother than on comparable screens in LCD or OLED (Organic Light Emitting Diode) technology.

6. SUMMARY

This article discusses the issue of quantum dots and covers their physical basics, characteristics, applications, and various related research. The article addresses how spatial confinement affects the properties of electrons in nanoscale materials, transforming them into quantum dots with a discrete energy distribution. It touches on the use of quantum dots in technologies such as single-electron transistors, photodetectors, light-emitting diodes (QLED), and in medicine as cancer markers. It also discusses advanced fundamental research, including effects such as the Coulomb blockade, the Kondo effect, Andreev reflections, and potential applications in quantum cryptography and as logic gates in quantum logic. The article presents both technical details and the current state of knowledge about quantum dots, their applications, and potential future research directions.

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