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SELECTED ERROR COMPENSATION PROBLEMS OF THE AHRS/INS CARDANLESS INERTIAL NAVIGATION SYSTEM

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Abstract
The paper presents algorithms for compensating selected errors of the AHRS/INS system at various stages of its construction and testing. The object of the research was a laboratory model of the system built at the Air Force Institute of Technology. Methods of identifying parameters of measurement signals and Kalman filtering are presented. Methods of compensation of instrumental errors related to the mounting geometry of measuring sensors used in the system were discussed. Problems related to the compensation of stochastic errors in the measurement circuits were also presented. One of the more difficult problems is the compensation of the BIAS error and the compensation of the non-commutativity of rotations around the measurement axes. Accurate compensation of errors in cardanless inertial navigation systems has a large impact on the accuracy of determining the attitude in AHRS systems and the accuracy of determining the position in INS systems.
Keywords*: algorithms, error compensation, Inertial Navigation System*

WYBRANE PROBLEMY KOMPENSACJI BŁĘDÓW BEZKARDANOWEGO SYSTEMU NAWIGACJI INERCJALNEJ AHRS/INS

Streszczenie
W artykule przedstawiono algorytmy kompensacji wybranych błędów systemu AHRS/INS na różnych etapach jego konstruowania i testowania. Obiektem badań był model laboratoryjny systemu zbudowany w Instytucie Technicznym Wojsk Lotniczych. Przedstawiono metody identyfikacji parametrów sygnałów pomiarowych oraz filtrację Kalmana. Omówiono zastosowane w systemie sposoby kompensacji błędów instrumentalnych związanych z geometrią montażu czujników pomiarowych. Przedstawiono również problemy związane z kompensacją błędów stochastycznych występujących w torach pomiarowych. Jednym z trudniejszych problemów jest kompensacja błędu BIAS oraz kompensacja nieprzemienności obrotów wokół osi pomiarowych. Dokładna kompensacja błędów występujących w bezkardanowych systemach nawigacji inercjalnej ma duży wpływ na dokładność wyznaczania kątowego położenia przestrzennego w systemach AHRS oraz dokładność wyznaczenia pozycji w systemach INS.
Słowa kluczowe*: algorytmy, kompensacja błędów, System Nawigacji Inercjalnej*

1. Introduction

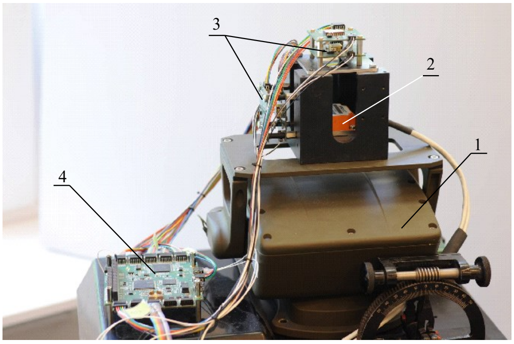
The fundamental problem in constructing AHRS/INS cardanless inertial navigation systems is error compensation[1], which must occur at every stage of its construction. These errors can be broadly divided into two types. The first comes from measuring sensors, which depends on the sensor class. When selecting a sensor, we can choose the sensor with the proper accuracy of determining parameters in the developed system. A situation where we design and perform measurement activities ourselves is a separate problem. In our solutions, we deal with the second source of errors that come from information processing algorithms and the calculation of the final parameters of the AHRS/INS system. The main focus is on two errors, which caused us the most problems in designing a cardanless attitude reference system. The first is the BIAS error, which is both deterministic and stochastic. The second is the error occurring with complex, angular spatial motion. It is

[1] H. Mussof, J.H. Murphy, *Study of strapdown navigation attitiude algorithms.* AIAA Journal of Guidance, Control and Dynamics, 1995.

associated with non-holonomy, where non-commutative errors of rotations around the measurement axes are prevalent[2].

2. Research object

As part of the research statutory work, the research object was a laboratory model of the AHRS/INS cardanless inertial navigation system built at the Air Force Institute of Technology. The block diagram of the tested system on the laboratory stand is shown in Fig. 1.



1 – angular movement generator, 2 – STIM measurement module, 3 – GYPRO measurement modules, 4 – laboratory module of the AHRS/INS system

Fig. 1. Research object installed on a laboratory stand

Source: own study.

This module consists of the following 4 basic modules:

- Measurement signal maintenance module (MOS);
- Main calculation module (GMO);
- External communication module (MKZ);
- Power supply module (MZ).

3. Identification of measurement signal parameters

Parameter identification of measurement signals is an initial phase in the process of designing the cardanless inertial navigation system. It is of fundamental importance due to designing signal filtration systems and the compensation of measurement errors. The algorithms for identifying the measurement signal parameters are applied during initial spatial orientation of the aircraft using the measurement platform. This stage serves also as the system preparation for the proper operation. The following identified parameters of real measurement signals were applied in the evaluation algorithms of the inertial navigation system:

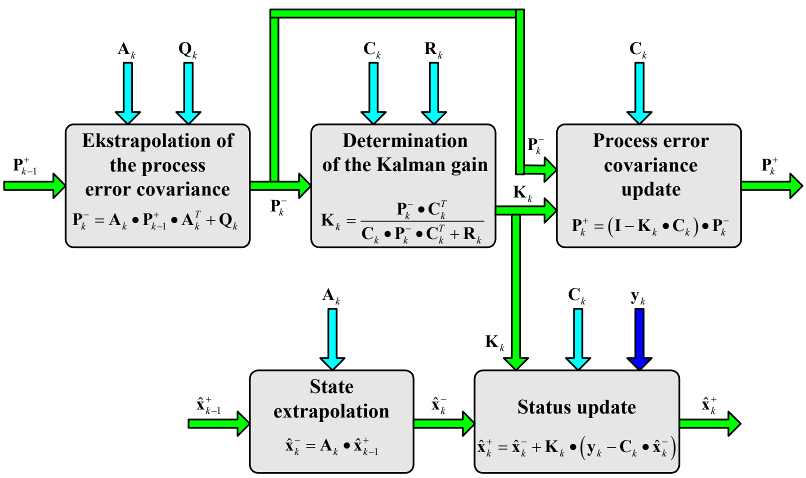
- Expected value of a random variable;
- Variance and standard deviation;
- Fast Fourier transform;
- Probability density function;
- Autocorrelation function;
- Autocovariance function;
- Random Walk function;

[2] I.Y. Bar-Itzhack, *Symbolic representation of translatory motion in multivarying-link mechanisms*, AIAA Journal of Guidance, Control and Dynamics, 1983.

- Attractor function on the phase plane;
- Change trend function;
- Median of a random variable – sevcond quartile;
- Position and velocity error on the phase plane.

4. Disturbance filtering of measurement signals

The problem with disturbance filtering of measurement signals is one of the most important for the proper operation of the cardanless inertial navigation system. Manufacturers of measurement sensors often introduce signal-filtering algorithms into the internal structure. Sometimes, however, they are not selected in a way that satisfies the AHRS/INS system designer due to a different function of the goal we want to achieve. The selection of the filter structure and algorithmization of calculations should include obtaining the appropriate filtration level and reducing dynamic errors which impact a wide range of system operating parameters. The Kalman filter is often applied due to wide possibilities and its flexibility. It was designed in 1960 by an American electrical engineer Rudolf Emil Kalman. It has been modified for many years, and now this filter has many variants. Moreover, is a very good literature position[3] taking into account theoretical and practical aspects. To filter signals in the AHRS/INS system, in the first approximation, the classical Kalman filter system can be used, assuming that there is no control but only measurement. The structural diagram of such an algorithm is shown in Fig. 2.



$\mathbf{x_k}$ – state vector, $\mathbf{A_k}$ – state matrix, $\mathbf{y_k}$ – output vector, $\mathbf{C_k}$ – output matrix, $\mathbf{P_k}$ – process error covariance error, $\mathbf{Q_k}$, $\mathbf{R_k}$ – matrices depending on the variance of disturbance random variable for process noise and measurement noise

Fig. 2. Structural diagram of the algorithm for Kalman filtration

Source: own study.

In this algorithm, a number of simplifying assumptions are made regarding process noise, measurement noise, and initial conditions for starting the algorithm must be determined. Note that matrices A and C should be updated at each step because the process is non-stationary. The elements of these matrices change over time and should be identified. This also applies to the Q and R matrices, whose elements depend on the variance of the random disturbance variable which fluctuates in time.

[3] S.G. Mohinder, P.A. Angus, *Kalman filtering, theory and practice using Matlab*, John Wiley & Sons, 2001.

5. Compensation of geometric errors of measurement sensors

Among the geometric errors of measuring sensors, three types can be distinguished: rotation error, skew error and module error. Two of them are illustrated in Fig. 3.

Rotation error occurs if the system $x_1^{'}, x_2^{'}$ is rotated relative to the system by an angle, but the axes of the systems are perpendicular to each other. The skew error occurs when the axes are rotated in opposite directions and are not perpendicular to each other. The third error, the modulus error, occurs when the vector module before transformation is not equal to the vector module after transformation.

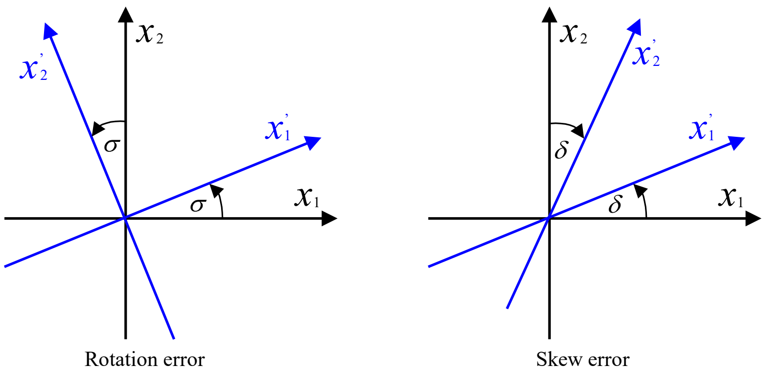


Fig. 3. Geometric errors of locating the axis of measurement sensors
Source: own study.

Many authors have dealt with the problem of eliminating geometrical errors, e.g.[4]. Eliminating all geometric errors leads to complex algorithms, which should determine one transformation matrix:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{MT} \cdot \begin{bmatrix} x_1^{'} \\ \vdots \\ x_3^{'} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1^{'} \\ \vdots \\ x_3^{'} \end{bmatrix} \tag{1}$$

Manufacturers of measurement modules strive to minimize geometric errors. The smallest errors might occur if the measurement module is made as one cohesive structure. Exemplary values of the transformation matrix coefficients after performing the program procedure of reducing the geometric errors are as follows:

$$\mathbf{MT_1} = \begin{bmatrix} 9.9941330e\text{-}01 & -5.8932617e\text{-}04 & 5.8000000e\text{-}04 \\ -5.8000000e\text{-}04 & 9.9940936e\text{-}01 & 5.8000000e\text{-}04 \\ 5.8000000e\text{-}04 & 5.8000000e\text{-}04 & 9.9942000e\text{-}01 \end{bmatrix}$$

$$\mathbf{MT_2} = \begin{bmatrix} 8.5028000e\text{-}01 & -1.4972000e\text{-}01 & 1.4972000e\text{-}01 \\ -3.1813000e\text{-}01 & 6.8186000e\text{-}01 & 3.1813000e\text{-}01 \\ 5.1201000e\text{-}01 & 5.1201000e\text{-}01 & 4.8799000e\text{-}01 \end{bmatrix}$$

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[4] Z. Gosiewski, A. Ortyl, *Algorytmy inercjalnego bezkardanowego systemu orientacji i położenia obiektu o ruchu przestrzennym*, Instytut Lotnictwa, Warszawa 1999, p. 74–79; A. Ortyl, Z. Gosiewski, *Porównanie optymalnych algorytmów ortogonalizacji macierzy i normalizacji kwaternionów*, Zeszyty Naukowe Politechniki Rzeszowskiej, Mechanika, Awionika, 1998; Y.F. Jiang, Y.P. Lin, *Improved strapdown coning algorithms*, IEEE Transactions on Aerospace and Electronic Systems, 1992.

The first transformation matrix concerns a measurement module with a compact structure and good accuracy class purchased from a commercial company. The second matrix conserns the measurement module we built before precise, mechanical setting of the measurement axes. Ideally, when there are no geometric errors in the measurement module, the transformation matrix is an identity matrix. The research shows that regardless of whether a ready-made set of a measurement module is purchased, or done, a procedure for eliminating geometric errors should be performed.

6. Compensation of bias of measurement sensors

The BIAS error is usually understood as the constant component of the measurement signal. In this article, this concept will be expanded. It will be understood as a determined part of the total error, including the constant component of the signal and the variable component, taking into account the trends of signal changes over time with the unchanged value of the measured signal. BIAS error compensation is critical to the accuracy of AHRS/INS systems. Identifying BIAS and its compensation is one of the most difficult tasks when building a system. It is difficult since apart from the determined part of this error, there is also a stochastic part. The constant component is subject to stochastic changes depending on time, temperature and other, not fully defined factors. Moreover, there are components defined as trends of changes over time. The factor that further complicates the problem is the fact that turning on the measurement system, identifying the BIAS parameters, and then turning it off and on again with re-identification gives different identification results.. Therefore, in AHRS/INS systems, there is a stage of initial angular orientation and spatial position in which such identification is made. After the initial orientation stage, the system is not turned off. Still, it goes to the actual operation stage, in which the BIAS is corrected based on the identified parameters. This situation is illustrated in Fig. 4.

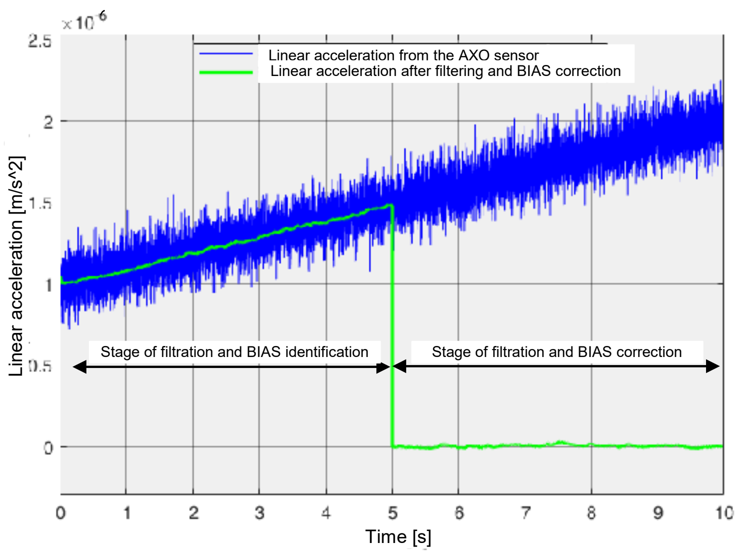

Fig. 4. An example of illustrating the effectiveness of Kalman filtration and BIAS error correction for the a_x component of linear acceleration
Source: own study.

The actual measurement signal obtained from the linear acceleration sensor has a constant component, a certain tendency of changes over time and is disrupted. In fact, the measurement sensor of the acceleration component is motionless. The constant component should be zero and with no change trend. In the initial stage, the Kalman filtering of the signal and the identification of the constant component and the trend of changes occur. Filtering and BIAS correction based on the identified parameters occur in the next stage. In this example, the BIAS of the acceleration component in the *0x* axis was successfully compensated to $10^{-8}\ \mathrm{m \cdot s^{-2}}$.

7. Compensation of non-holonomic errors

Errors related to non-commutativity of rotations during angular velocity measurements for the purposes of determining Euler angles, often called non-holonomic errors, were observed in the 1950s when there was rapid development in the production and testing of gyroscopic instruments. At that time, one of the unexpected results was the hitherto famous phenomenon of error formation in conical motion. This phenomenon has only just been clarified in 1958 by Goodman and Robinson[5].

Non-holonomic errors can be mitigated in two ways. First, by increasing the sampling frequency of gyroscope measurement signals for any angular motion, we should take into account the Shannon sampling theorem. The second way is to introduce additional compensation signals that limit these errors. The first method has major limitations since the sampling frequencies for speed gyroscopes manufactured today are usually from 50 Hz to 2000 Hz, which may not be sufficient for this method. The second method, related to compensation signals, was first suggested by John E. Bortz[6]. The Bortz method is a classic method that has undergone many modifications and has been an inspiration to develop new methods[7]. The basis for this method is a vector equation describing the derivative of the spatial orientation vector:

$$\dot{\vec{\phi}}(t) = \vec{\omega}(t) + \dot{\vec{\sigma}}(t), \tag{2}$$

by initial conditions: $\vec{\phi}(t_0) = \vec{0}$, where: $\dot{\vec{\phi}}(t)$ – the time derivative of the spatial orientation vector, $\vec{\omega}(t)$ – angular velocity vector, measured with inertial methods (e.g. by velocity gyroscopes), $\vec{\omega} = [p, q, r]^T$ – vector of angular velocities of the system associated with the measurement platform, relative to the reference system (inertial), $\dot{\vec{\sigma}}(t)$ – non-commutative vector of angular velocity (non-measurable with inertial methods).

Non-commutative vector of angular velocity is described as follows:

$$\dot{\vec{\sigma}}(t) = \frac{1}{2}\vec{\phi}(t) \times \vec{\omega}(t) + A[\phi(t)] \cdot \vec{\phi}(t) \times \left[\vec{\phi}(t) \times \vec{\omega}(t)\right] \tag{3}$$

where:

$$A[\phi(t)] = \frac{1}{\phi^2(t)} \cdot \left[1 - \frac{\phi(t) \cdot \sin[\phi(t)]}{2 \cdot \{1 - \cos[\phi(t)]\}}\right],$$

$$\vec{\phi}(t) = [\phi_x(t), \phi_y(t), \phi_z(t)]^T, \quad \phi(t) = \sqrt{\phi_x^2(t) + \phi_y^2(t) + \phi_z^2(t)}.$$

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[5] L.E. Goodman, A.R. Robinson, *Effects of finite rotations on gyroscope sensing devices*, J. Appl. Mech., vol. 25, June 1958.

[6] J.E. Bortz, *A new concept in strapdown inertial navigation*, NASA TR R-329, 1970.

[7] M.B. Ignagni, *On the orientation vector differential equation in strapdown inertial systems*, IEEE Transactions on Aerospace and Electronic Systems, 1994; Y.F. Jiang, Y.P. Lin, *On the rotation vector differential equation*, IEEE Transactions on Aerospace and Electronic Systems, 1991; M.B. Ignagni, *Optimal strapdown attitude integration algorithms,* AIAA Journal of Guidance, Control and Dynamics, 1990.

Theoretically, the most undesirable motion for AHRS algorithms is the motion, in which the vector $\vec{\omega}(t)$ is perpendicular to vector $\vec{\phi}$, so: $\vec{\phi}(t) = \int_0^t \vec{\omega}(t) \cdot dt$. Such a motion that partially satisfies the condition of perpendicularity is the so-called conical movement. This motion can be described by the relation of vector components of the angular velocity measured by inertial methods:

$$\vec{\omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \Omega_s \cdot \sin(\alpha_s) \cdot \cos(\Omega_s \cdot t) \\ -\Omega_s \cdot \sin(\alpha_s) \cdot \sin(\Omega_s \cdot t) \\ \Omega_s \cdot [1 - \cos(\alpha_s)] \end{bmatrix} \tag{4}$$

which corresponds to the spatial orientation vector in the following form:

$$\vec{\phi} = \begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} = \begin{bmatrix} \alpha_s \cdot \sin(\Omega_s \cdot t) \\ \alpha_s \cdot \cos(\Omega_s \cdot t) \\ 0 \end{bmatrix} \quad \text{or} \quad \dot{\vec{\phi}} = \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{bmatrix} = \begin{bmatrix} \alpha_s \cdot \Omega_s \cdot \cos(\Omega_s \cdot t) \\ \alpha_s \cdot \Omega_s \cdot \sin(\Omega_s \cdot t) \\ 0 \end{bmatrix} \tag{5}$$

Conical motion results in some of the greatest errors in determining the angular spatial orientation vector.

The classic Bortz algorithm does not provide satisfactory accuracy results and elimination of non-commutative errors. It was modified by the authors by internal control of the accuracy of the solution of differential equations. This problem boils down to finding the zero of the error function.

The error function was determined on the basis of the algebraic form of the differential vector equation (2):

$$\begin{aligned} DF_x &= \frac{d\phi_x(t)}{dt} - p(t) - \dot{\sigma}_x(t) + \delta_x(t) = 0 \\ DF_y &= \frac{d\phi_y(t)}{dt} - q(t) - \dot{\sigma}_y(t) + \delta_y(t) = 0 \\ DF_z &= \frac{d\phi_z(t)}{dt} - r(t) - \dot{\sigma}_z(t) + \delta_z(t) = 0 \end{aligned} \tag{6}$$

where: DF_z, DF_y, DF_z – components of error function in *x*, *y*, *z* axes of navigation system,

$\delta_x, \delta_y, \delta_z$ – additional correction signals resetting the error function,

$\dot{\sigma}_x = 0,5 \cdot w_{1x} + A \cdot w_{2x}$, $\dot{\sigma}_y = 0,5 \cdot w_{1y} + A \cdot w_{2y}$, $\dot{\sigma}_z = 0,5 \cdot w_{1z} + A \cdot w_{2z}$

$w_{1x} = \phi_y \cdot r - \phi_z \cdot q$, $w_{1y} = \phi_z \cdot p - \phi_x \cdot r$, $w_{1z} = \phi_x \cdot q - \phi_y \cdot p$,

$w_{2x} = \phi_y \cdot (\phi_x \cdot q - \phi_y \cdot p) - \phi_z \cdot (\phi_z \cdot p - \phi_x \cdot r)$, $w_{2y} = \phi_z \cdot (\phi_y \cdot r - \phi_z \cdot q) - \phi_x \cdot (\phi_x \cdot q - \phi_y \cdot p)$,

$w_{2z} = \phi_x \cdot (\phi_z \cdot p - \phi_x \cdot r) - \phi_y \cdot (\phi_y \cdot r - \phi_z \cdot q)$.

Internal error correction eliminates more accurately both non-commutative errors in rotation and numerical errors associated with delays in algorithms. The results obtained in the classic Bortz algorithm were compared with the algorithm modified by the internal error correction. The block diagram of the comparison is exhibited in Fig. 5.

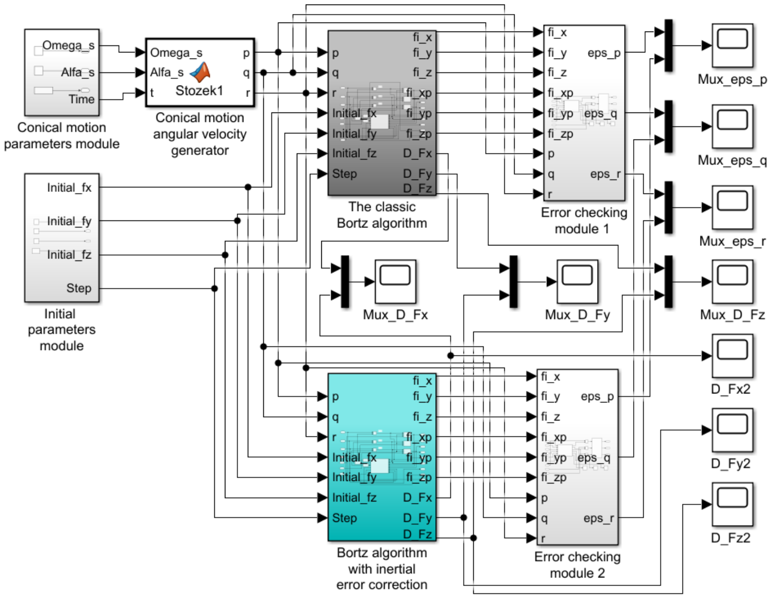

Fig. 5. A block diagram of comparison of the classic Borth algorithm with the algorithm of internal correction
Source: own study.

Exemplary comparison results of the operation of algorithms are shown in Fig. 6.

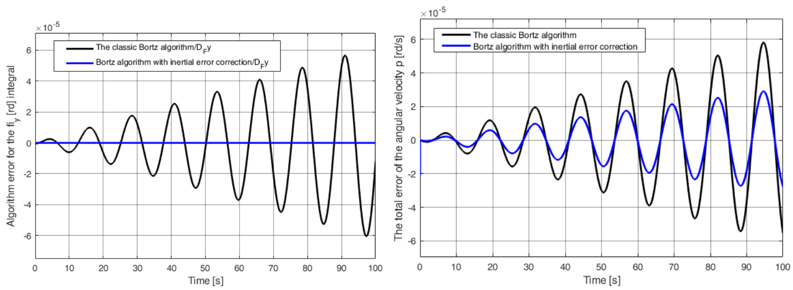

Fig. 6. Exemplary comparison results of errors for the classic and Bortz algorithm with internal correction
Sourcd: own study.

The presented graphs show that the internal error correction reduces the error rate of the algorithm, but the values for the total angular velocity error are still too large, despite the significant improvement. This result was the reason for the subsequent modification of the Bortz algorithm, in which the external error correction module was introduced as feedback to the set values p, q, r. The block diagram of this modification is shown in Fig. 7.

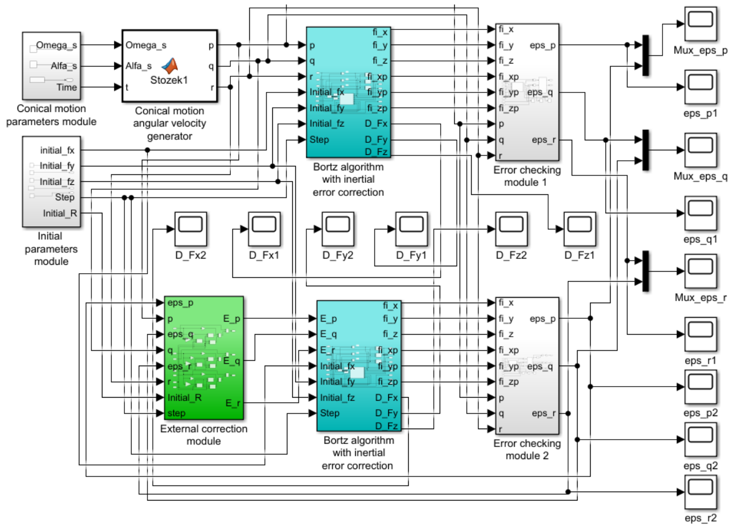

Fig. 7. Block diagram of comparing an algorithm with internal and external error correction
Source: own study.

The external correction module implements the inverse Bortz algorithm given in the form of a vector differential equation:

$$\vec{\omega} = \dot{\vec{\phi}} - \frac{1-\cos(\phi)}{\phi^2} \cdot \vec{\phi} \times \dot{\vec{\phi}} + \frac{1}{\phi^2} \cdot \left[1 - \frac{\sin(\phi)}{\phi}\right] \cdot \vec{\phi} \times \left(\vec{\phi} \times \dot{\vec{\phi}}\right) \tag{7}$$

and controller algorithm P, I, I^2, where: P – proportional part of the controller, I – integral part of the controller, I^2 – double-integral part of the controller.

The results of two modernized Bortz algorithms, which are shown in Fig. 8 and 9, were compared.

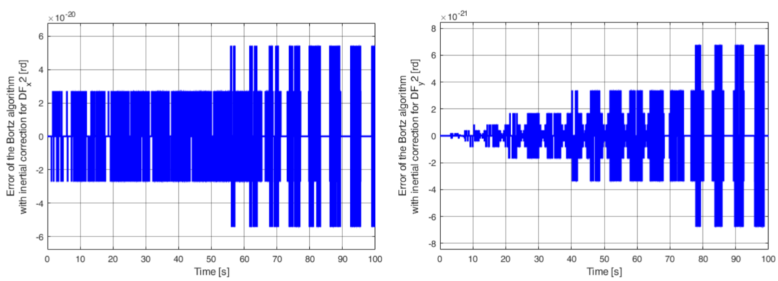

Fig. 8. Exemplary numerical errors of the Bortz algorithm with internal correction for *0x* and *0y* axis
Source: own study.

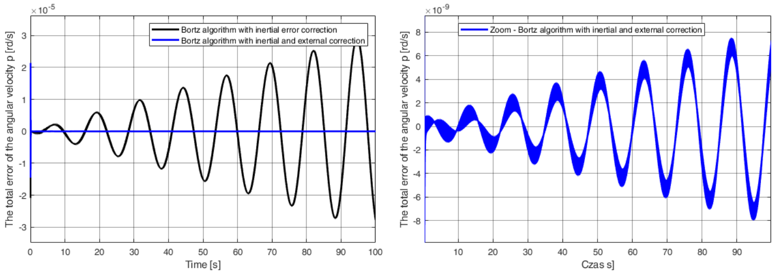

Fig. 9. The exemplary total error of the angular velocity *p* for the Bortz algorithm with internal correction and with internal and external correction

Source: own study.

The presented graphs show that the numerical errors of the modified Bortz algorithm are satisfactory. The increasing numerical errors resulting from the physics of phenomena will be at an acceptable level in the AHRS/INS operating time range. As a result of using the external error correction, the level of errors of the total angular velocities p, q, r was reduced by several orders of value compared to the classic Bortz algorithm modified with internal error control.

8. Conclusions

1. At each stage of constructing the AHRS/INS cardanless inertial navigation system, a thorough analysis of errors and corrections should be made to minimize errors to a level acceptable from the point of view of system accuracy.
2. Geometric errors of the measurement axes of the measurement sensors can be minimized or eliminated by precise calculation of the transformation (correction) matrix.
3. BIAS errors are one of the most difficult to minimize due to their stochastic nature, the parameters of which may undergo unpredictable changes over time.
4. Kalman filtration used in algorithms must have algorithms for process identification and parameter correction in each calculation step.
5. Rotation non-commutativity errors and numerical errors, despite their increasing nature, can be reduced by several levels compared to the classic Bortz algorithm, thanks to introducing two additional error correction methods.

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